Mustang Math Tournament 2024

Herding Hexes Stallion Round

Basic Format

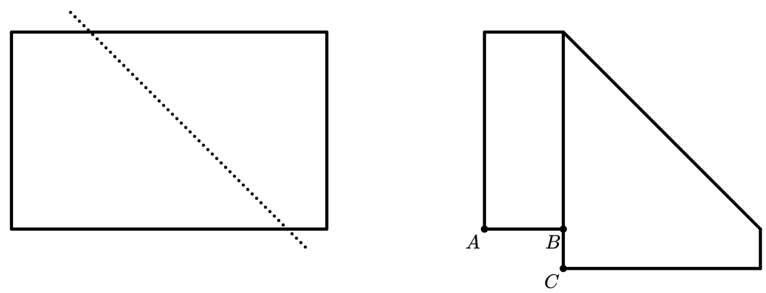
- This round contains 26 problems to be solved in 45 minutes.
- Each problem corresponds to a hexagon on the answer grid (backside).
- A correct answer will grant 2 points each for Problems 1 through 10, 3 points each for Problems 11 through 19, and 4 points each for Problems 20 through 26.
- The score of a hexagonal tile is doubled if it can be connected back to the Free tile through other tiles that contain correct answers.
- **Do not** write below the provided answer blank inside each hexagon (the space is for grading purposes).
- *Feel free to **flag** down a proctor if you need help **deciphering** any of the above instructions*

This page left blank intentionally. You may use this page for scratch work.

WE WILL NOT GRADE ANY WORK ON THIS PAGE.

1. A small ice cream truck offers vanilla cones and chocolate cones. A customer can add sprinkles, caramel, both, or neither. How many different cones can be ordered at the truck?

2. A rectangular piece of paper shown on the left is folded along a diagonal line, and the resulting figure is shown on the right. If the original rectangle is 5 inches by 8 inches and the length of AB is 2 inches, and $\angle ABC$ is a right angle, what is the length of BC?



3. Find the sum of the real numbers x that satisfy the equation $x^2 - 65 = 16$.

4. The number 2024 has a prime factorization of $2 \times 2 \times 2 \times 11 \times 23$. Suppose that a, b, c, d, and e are five distinct positive integers such that $a \times b \times c \times d \times e = 2024$. What is the sum $a + b + c + d + e$?

5. Let a, b, and c be integers greater than 1 such that $\gcd(b, c) = \gcd(c, a) = \gcd(a, b) = 1$ and $abc = 34300$. What is the positive difference between the greatest and least numbers among a, b, and c?

6. Suppose x and y are real numbers such that $(x + 2y)^2 = 9$ and $x + 3y = 10$. What is the sum of all possible values of x?

7. Farmer John has n glasses and a bucket filled with 1 gallon of milk. He pours half of his bucket in the first glass, and then half of what remains in his bucket in the second glass, and so on until the nth glass. Once he reaches the last glass, he will cycle back and pour half of his bucket into the first glass again, continuing this process forever.

   Given that the ratio between the amounts in the largest and smallest glasses is $32 : 1$, find the number of glasses.

8. In the sequence of integers below, each number (other than the first and last) is less than the sum of its two neighbors. For example, the neighbors of 11 are 3 and 10, and $11 < 3 + 10$. What is the missing number N?

   $3, 11, 10, 2, N, 5, 0$

9. Let A and B be digits such that the five-digit number $A123B$ is a multiple of 5, and the four-digit number $B1A2$ is a multiple of 11. What is the maximum possible value of $A + B$?

10. Suppose N is a perfect square which has 3 digits. Given that the first and last digits are equal, what is the largest possible value of N?

11. Susan chooses a random integer between 10 and 99, inclusive. What is the probability that it is a perfect square?

12. The Awesome Intellectual Mustangs Examination is a 10-problem test, and each problem falls into one of four categories: algebra, combinatorics, galloping, and neighing. Among any three consecutive problems, there must be three different categories represented. How many different tests are possible? (Two tests are considered distinct if their sequences of problem categories are distinct.)

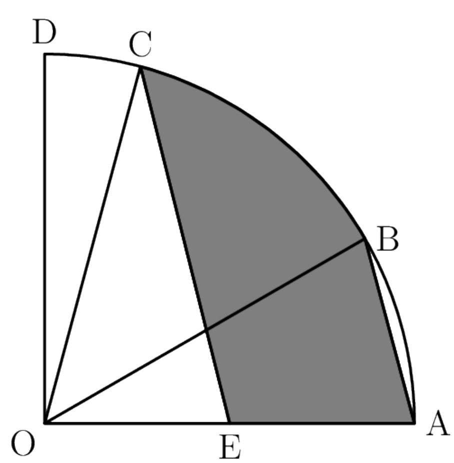
13. Let $n > k$ be positive integers. Alice is initially tasked with selecting a k-person committee from among n club members. If the committee size were instead to be $k+1$, Alice would have twice as many ways to select the committee as before. If the committee size were instead to be $k-1$, the number of ways to select the committee would be one third the original amount. Find the number of ways for Alice to select the k-person committee.

14. Evaluate the following epxression:

$$\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\cdots+\left(\frac{1}{100}+\frac{2}{100}+\frac{3}{100}+...+\frac{99}{100}\right).$$

15. When 20240000^2 is divided by 81, the remainder is 73. What is the remainder when 20240000 is divided by 81?

16. A quarter circle centered at O with radius 2 is shown in the diagram below. If $\angle AOB = 30°$, $\angle BOC = 45°$, and $CE \parallel AB$, find the area of the shaded region.



17. Let A, B, and C be distinct integers such that A, B, C forms an increasing arithmetic sequence, in that order, and A, C, B forms a geometric sequence, in that order. What is the minimum possible value of C?

18. If x, y are positive integers not exceeding 100, compute the maximum possible value of
$$\operatorname{lcm}\left(\gcd\left(x,y\right),\operatorname{lcm}\left(x,y\right)\right).$$

19. Let ABC a triangle with $AB = 5$, $BC = 12$, and $\angle ABC = 90^\circ$. A circle whose center lies on side AC is tangent to sides AB and BC. Find the radius of the circle.

20. Define the polynomial $P(x) = x^3 + 1$. Find the sum of the coefficients of $P(P(P(x)))$.

21. Suppose that p, q, r, s, t is an increasing arithmetic sequence such that $p+q+r+s+t = 25$ and $pqrst = 395$. What is the sum of all possible values of pt?

22. Let $ABCD$ be a square with side length 4. A point P is randomly chosen in the interior of $ABCD$ and a circle ω with radius 1 is drawn with center at P. What is the probability ω intersects the boundary of square $ABCD$ at exactly four points?

23. Let Ω be a circle with diameter AB of length 10. Let C be a point outside Ω, such that segment BC intersects Ω again at D. Suppose that AC and CD have integer lengths and $\angle ABC = 60^\circ$. What is the sum of the possible values of AC?

24. A broken calculator currently displays the number 44. It has two buttons: pressing the first will replace the display number n with $2n - 39$, whereas pressing the second will add 1 to the display number. What is the minimum number of button presses needed to make the calculator display the number 2024?

25. Let A and B be points on a plane that are 1 unit apart. Consider the set of points C on the plane such that $30^\circ < \angle ACB < 60^\circ$. If the area of this set is written as $a\pi + b\sqrt{3}$ for rational numbers a and b, find $a + b$.

26. Let a special triangle be one that has consecutive integers for side lengths, and an integer area. For example, a triangle with side lengths of 3, 4, and 5 is special because it has an area of 6. Find the sum of the perimeters of the three smallest unique special triangles.

Acceptable Answers

The following rules provide guidelines for acceptable answers in this round. Please note that any specifications provided in a problem will take precedence over these rules. The decisions of MMT coordinators are final.

- Common fractions are defined as a fraction in the form $\pm\frac{a}{b}$ where a and b are natural numbers and $\gcd(a, b) = 1$.
- Ratios and fractional answers should be expressed as common fractions unless otherwise specified.
- Radicals should be simplified. A simplified radical must satisfy:
  - No square factors, fractions, or nested radicals inside a radical
  - No radicals inside the denominator of a fraction
- Answers must be expressed to the exact accuracy called for in the problem (e.g. 25.0 will not be accepted for 25 and 25 will not be accepted for 25.0).
- Do not make approximations for numbers (e.g. 3.14 or $\frac{22}{7}$ for π) unless otherwise specified.
- Units **do not** need to be included but must be correct if included.